Transient behavior of the photorefractive space-charge field

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(Received 1 June 1999; revised manuscript received 9 May 2000)

The transient behavior of the fundamental amplitude of the photorefractive space-charge field is derived without restrictions within the band transport model. The two-wave mixing pulse excitation regime is simulated in different ranges of the main parameters, especially in the region where the nonlinear processes become important. The formation and decay of the field are analyzed, and for different ranges of the excitation parameters the corresponding behaviors are predicted. Features of the field such as the nonlinear decay of the photorefractive field at high-writing pulses, its oscillatory behavior when an external field is applied, and the dependence of its formation and decay time on the grating spacing are studied.

I. INTRODUCTION

Since the photorefractive effect discovery,[1] the materials that present such phenomenon have been extensively studied in their many applications.[2,3] From a theoretical point of view, two different models are used to explain the photorefractive effect. The hopping model developed by Feinberg *et al.*[4] and the band transport model, called photogeneration-diffusion-drift-trapping (PDDT),[5] conceived by the Kiev group.[6,7] This is the most used model. There are numerous experimental results satisfactorily explained by the PDDT model. Among them we can mention the independence of the diffraction efficiency on the average excitation intensity in cw regime, the $\pi/2$-phase shift of the index grating with respect to the excitation pattern, and the holographic-current dynamic.[2,5,7–15] In addition, this model is utilized as background when some effects in photorefractive materials are analyzed, e.g., the study of sub-harmonic gratings,[16] the formation of solitons,[17] and the space-charge waves.[18] Sublinear conductivity dependence on the average excitation intensity,[19] multiple exponential decay in the photocurrent, diffraction efficiency,[20] and the formation of complementary gratings[21] are experimental results that were explained by more complex models. These models postulate additional hypotheses such as the presence of thermally excitable shallow photorefractive centers[22] or electron-hole transport mechanism.[23] However, it should be pointed out that these models do not imply new dynamical processes. They are extensions of the early Kukhtarev model based on photogeneration-diffusion-drift-recombination in traps. By using the PDDT model framework, the behavior of the space-charge field responsible for the photorefractive effect, was successfully predicted.[24–29] This model is described by a set of nonlinear and coupled equations that are called material or Kukhtarev equations. A literature review indicates that analytical solutions are obtained only when additional assumptions are added in order to reduce the complexity of the equations.[8,10,24] Nevertheless the validity range of these solutions are restricted to certain simplified regimes. In the search of solutions several numerical methods have been employed.[9,14,27–29] The disadvantage of the numerical solutions is that they can be unstable in a certain analyzed range with the risk of a wrong interpretation. Indeed, a general analytic solution has not yet been reported. The aim of this paper is finding a general analytic solution for the fundamental amplitude of the space-charge field from the PDDT model under two-wave mixing excitation setups permitting to analyze regions of interest. The attainment of such a solution can give us more reliable physical interpretations, especially when the nonlinear effects cannot be neglected. In this paper we analyze the transient behavior, i.e., formation and decay, of the photorefractive field simulating typical bismuth silicon oxide (BSO) material parameters.

II. SPACE-CHARGE FIELD EQUATION

The PDDT model postulates a photorefractive material with a photoexcitable impurity of concentration N_D, which originates a free charge carrier concentration n, involving either electrons or holes. At equilibrium, a certain fraction N_{Deq}^+ of photorefractive impurities is ionized. Moreover, the model assumes the existence of a concentration of nonactive ionized impurities of density $N_A = N_{Deq}^+$ that do not participate in the photorefractive process but ensure the neutrality of the charge in the crystal. Diffusion and drift transport mechanisms for the charge carriers are taken into account. When the absorption, and the two-wave coupling variation in the propagation direction z are disregarded, the Kukhtarev equations are reduced to one-dimensional equations in the x direction, perpendicular to the excitation direction z:[7]

$$\frac{\partial N_D^+}{\partial t} = (sI + \beta)(N_D - N_D^+) - \gamma n N_D^+ , \quad (1)$$

$$\frac{\partial E}{\partial x} = \frac{e}{\epsilon \epsilon_0}(n + N_A - N_D^+), \quad (2)$$

$$J = e\mu n E - k_B \mu T \frac{\partial n}{\partial x}, \quad (3)$$

$$\frac{\partial J}{\partial x}=-e\frac{\partial}{\partial t}(n+N_A-N_D^+), \tag{4}$$

with J the current density, $E=E_0+E_{sc}$ the total electric field with the external electric field E_0 included, s the cross section of photoionization, γ the recombination constant, μ the electron mobility, e the electron charge, β the thermal excitation rate, k_B the Boltzmann's constant, ϵ the dielectric constant, ϵ_0 the vacuum permittivity, T the temperature, and x the spatial coordinate perpendicular to the excitation direction. We assume that incident wave vectors describe an isosceles triangle with the grating vector $\mathbf{K}$ of modulus $K=2\pi/\Lambda$. The grating spacing is $\Lambda=\lambda/2\sin(\theta/2)$ where λ is the wavelength employed and θ is the angle between incident wave vectors. The steady-state fringe recording spatial pattern dependence is[2]

$$I=I_0[1+m\cos(Kx)], \tag{5}$$

where $I_0=(I_1+I_2)/2$ is the pulse writing average intensity, I_1 and I_2 are the incident beam intensities, and $m=2\sqrt{I_1I_2}/I_0$ is the modulation depth. If the light erasure regime is assumed, the excitation in Eq. (5) is $I=I_0$ because $m=0$. In typical experiments, the spacing Λ is much smaller than the crystal size. Then, it is possible to disregard boundary effects and the varying magnitudes can be treated as periodic, with Λ the period of the intensity pattern I. In the small modulation regime,[2,7,24] the solutions can be written in the form

$$\Psi(x,t)=\psi_0+\psi_1(t)\cos(Kx+\varphi_1), \tag{6}$$

where Ψ represents any of E, J, N_D^+, and n magnitudes. From the system (1)–(4), and extending Eq. (6) to the complex plane it is possible to obtain the equation for the temporal evolution of the space-charge field amplitude, E_1.[10,14–25] The equation written in its more general form is

$$\begin{aligned}\frac{d^2E_1}{dt^2}&+\left[sI_0+\beta+\gamma N_A\left(1+\frac{E_D+iE_0}{E_M}\right)\right.\\&\left.+(\gamma_0+2\gamma)n_0(t)\right]\frac{dE_1}{dt}+\left[\gamma_0(sI_0+\beta)(N_D-N_A)\right.\\&\left.+\gamma N_A\left(\frac{E_D+iE_0}{E_M}\right)[sI_0+\beta+\gamma n_0(t)]+\gamma_0\gamma n_0^2(t)\right]E_1\\&=imsI_0\frac{E_q}{E_M}(E_D+iE_0)\gamma[N_D-N_A-n_0(t)],\end{aligned} \tag{7}$$

with $E_D=Kk_BT/e$ the diffusion field, $E_q=eN_A/K\epsilon\epsilon_0$ the saturation field, $E_M=\gamma N_A/\mu K$, and $\gamma_0=e\mu/\epsilon\epsilon_0$. The free charge-carrier density average n_0 evolves satisfying the following nonlinear equation,[10,24]

$$\frac{dn_0}{dt}=(N_D-N_A)(sI_0+\beta)-(sI_0+\beta+\gamma N_A)n_0-\gamma n_0^2. \tag{8}$$

As mentioned above, Eq. (7) is derived in a simplified form in numerous papers.[9,10,14–26] Unfortunately, these simplified equations are applicable only in particular cases because of the use of additional restrictions in the physical picture. Valley[24] has investigated the evolution of E_1 in the very short pulse and in the quasi-cw regimes obtaining a version of Eq. (7) [Eq (47) in Ref. 24]. When the nonlinear recombination of free charge carriers and thermal rate of deep centers are disregarded analytical solutions have been obtained. In addition, the average electron density is assumed moderately low in his work. Jones and Solymar[9] have numerically solved the equivalent to Valley's equation in the short pulse case. They considered the nonlinear recombination of charge carriers but neglected the thermal rate of deep centers that is responsible for the grating decay to zero, in the dark. As a consequence a nonexisting grating remains permanently stored and the dependence of recording and storage time on grating spacing are not considered under this approximation. The thermal excitation is also neglected in other versions of Eq. (7), obtained by Au (Ref. 26) for the short pulse case. In addition, he assumes the total donor density N_D to be much larger than the ionized donor density N_D^+, which only occurs for a limited number of materials, and its analytical solution was obtained disregarding the E_1 terms, strongly limiting the validity range. Valley's version of Eq. (7) was also used by Okamura *et al.*[14] to describe space-charge field evolution under illumination with very short and weak mode-locked pulses, and a limited analytic solution was introduced in Okamura's work. On the other hand, both the thermal reexcitation of photorefractive centers and the nonlinear recombination of charge carriers are taken into account in the extensive theoretical-experimental paper of Fluck *et al.*[10] where Eq. (7) was obtained in complete form. However, in order to derive an analytical solution, they neglected the second-order derivative of the space-charge field that can produce a nonexisting dynamic of E_1 in various interesting ranges, in particular when high-power laser radiation is used. Summarizing, in the works mentioned above, only a partial analysis of the physical picture was done due to the lack of a general analytic solution of Eq. (7). Our aim is to derive this general solution.

III. GENERAL ANALYTIC SOLUTION

To solve Eq. (7) we need the free charge carrier density evolution $n_0(t)$, which is obtained by solving Eq. (8):

$$n_0(t)=N_A\frac{(n_{00}-N_An_-)n_+-(n_{00}-N_An_+)n_-\exp[-a(t-t_0)]}{(n_{00}-N_An_-)-(n_{00}-N_An_+)\exp[-a(t-t_0)]}, \tag{9}$$

where the initial density of free charge carriers n_{00}, can be deduced from the state of the system at time t_0. The adimensional parameters $n_\pm$ are defined as in Ref. 15

$$n_\pm = \frac{-(sI_0+\beta+\gamma N_A)\pm a}{2\gamma N_A}, \tag{10}$$

and

$$a=\sqrt{(sI_0+\beta+\gamma N_A)^2+4\gamma(sI_0+\beta)(N_D-N_A)}. \tag{11}$$

To obtain an analytic solution of Eq. (7), we apply successive transformations with the aim of obtaining an equation of well-known solutions. We define an auxiliary variable

$$\xi = \frac{(n_{00}-N_A n_+)}{(n_{00}-N_A n_-)}\exp[-a(t-t_0)], \tag{12}$$

expressing n_0 in terms of ξ as

$$n_0 = N_A(n_+ - \xi n_-)/(1-\xi) \tag{13}$$

with $\xi \neq 1$. Then, we introduce the function

$$W_1(\xi)=E_1(t)/mE_M, \tag{14}$$

and by replacing in Eq. (7) we obtain the following equation:

$$\xi^2(1-\xi)^2\frac{d^2W_1}{d\xi^2}+\xi(1-\xi)(b_1-b_2\xi)\frac{dW_1}{d\xi} + (b_3-b_4\xi+b_5\xi^2)W_1 = b_6+b_7\xi+b_8\xi^2, \tag{15}$$

where b_i are constants. We note that Eq. (15) could become a hypergeometric equation, which has well-known solutions. To this goal, we apply the transformation

$$(1-\xi)W_1 = W_2+\xi c_1+c_2, \tag{16}$$

where c_1 and c_2 are chosen so that the right-hand side of Eq. (15) becomes constant and the coefficient of W_2 turns out to be of degree one, obtaining

$$\xi^2(1-\xi)\frac{d^2W_2}{d\xi^2}+\xi(d_1-d_2\xi)\frac{dW_2}{d\xi}+(d_3-d_4\xi)W_2=d_6, \tag{17}$$

where d_i are constants. Equation (17) is a hypergeometric generalized equation, which has as particular solution a generalized hypergeometric series ${}_3F_2$,[31] so that

$$W_2^p = C\times {}_3F_2(1-\alpha_+,1-\alpha_-,1;1-\mu_+,1-\mu_-;\xi), \tag{18}$$

with C, $\alpha_\pm$, and $\mu_\pm$, constants depending on d_i. Our aim is to find the general solution of Eq. (17). Providing the appropriate initial conditions, we can analyze different regimes of interest. The general solution W_2 can be written as

$$W_2 = W_2^p + W_2^{gh}, \tag{19}$$

with W_2^{gh} the general solution of the Eq. (17) and $d_6=0$. To find W_2^{gh} we apply the transformation

$$W_2^{gh} = \xi^\delta W_3, \tag{20}$$

and obtain

$$\xi(1-\xi)\frac{d^2W_3}{d\xi^2}+(h_1+h_2\xi)\frac{dW_3}{d\xi}+h_3W_3=0, \tag{21}$$

with δ adequately chosen and h_i constants. The general solution of Eq. (21) is given by the linear combination of two hypergeometric series[31]

$$W_3 = A_1F(\eta_+,\eta_-;h_1;\xi)+A_2\xi^{-\varepsilon N_A/a} \times F(\eta_+-\varepsilon\gamma N_A/a,\eta_--\varepsilon\gamma N_A/a;2-h_1;\xi), \tag{22}$$

where A_i are constants determined by the initial conditions and $\eta_\pm$, and ε are constants that depend on h_i. Therefore, the general solution of Eq. (17) is finally obtained as

$$W_2^{gh} = C\times {}_3F_2(1-\alpha_+,1-\alpha_-,1;1-\mu_+,-\mu_-;\xi) + A_1\xi^{\delta_+}F(\eta_+,\eta_-;h_1;\xi) + A_2\xi^{\delta_-}F(\eta_+-\varepsilon\gamma N_A/a,\eta_--\varepsilon\gamma N_A/a;2-h_1;\xi). \tag{23}$$

From expressions (14), (16), (18), (19), and (23), $E_1(t)$ is then easily derived as the general solution of Eq. (7), giving

$$\begin{aligned} E_1(t) = {} & mE_M\left[1-\frac{(n_{00}-N_An_+)}{(n_{00}-N_An_-)}\exp[-a(t-t_0)]\right]^{-1}\Bigg\{c_2+c_1\frac{(n_{00}-N_An_+)}{(n_{00}-N_An_-)}\exp[-a(t-t_0)] \\ & +A_1\frac{(n_{00}-N_An_+)^{\delta_+}}{(n_{00}-N_An_-)^{\delta_+}}\exp[-a\delta_+(t-t_0)]F\left(\eta_+,\eta_-;h_1;\frac{(n_{00}-N_An_+)}{(n_{00}-N_An_-)}\exp[-a(t-t_0)]\right) \\ & +A_2\frac{(n_{00}-N_An_+)^{\delta_-}}{(n_{00}-N_An_-)^{\delta_-}}\exp[-a\delta_-(t-t_0)]F\left(\eta_+-\varepsilon\gamma N_A/a,\eta_--\varepsilon\gamma N_A/a;2-h_1;\frac{(n_{00}-N_An_+)}{(n_{00}-N_An_-)}\exp[-a(t-t_0)]\right) \\ & +C\times {}_3F_2\left(1-\alpha_+,1-\alpha_-,1;1-\mu_+,1-\mu_-;\frac{(n_{00}-N_An_+)}{(n_{00}-N_An_-)}\exp[-a(t-t_0)]\right)\Bigg\}, \end{aligned} \tag{24}$$

where A_1 and A_2 are obtained from the values of $E,dE/dt$ at time t_0. These values depend on previous excitation conditions. Constants appearing in Eq. (24) are expressed as functions of excitation and material parameters

$$c_1=\frac{isI_0E_q(E_D+iE_0)[N_D-N_A(1+n_-)]}{N_AE_M[an_-Eq-(E_D+iE_0)(sI_0+\beta+\gamma N_An_-)]}, \tag{25}$$

$$c_2=\frac{c_1\gamma N_A(sI_0+\beta+\gamma N_An_-)+i\gamma_0 sI_0(N_D-N_A)[2+(sI_0+\beta)/\gamma N_A]}{\gamma N_A(sI_0+\beta+\gamma N_An_+)}, \tag{26}$$

$$\varepsilon=\frac{1}{E_M}\sqrt{(E_qn_+-aE_M/\gamma N_A)^2+(E_D+iE_0)^2+2(E_D+iE_0)[E_qn_+-(sI_0+\beta)/\gamma N_AE_M+E_M]}, \tag{27}$$

$$\delta_\pm=\frac{\gamma N_A}{2a}\left[\left(\frac{E_D+iE_0}{E_M}\right)+\frac{E_q}{E_M}n_++\frac{a}{\gamma N_A}\pm\varepsilon\right]=\mu_\pm, \tag{28}$$

$$h_1=1+\frac{\varepsilon\gamma N_A}{a}, \tag{29}$$

$$\eta_\pm=\frac{E_q}{2E_M}+\frac{\varepsilon\gamma N_A}{2a}\pm\frac{\gamma N_A}{2a}\sqrt{[a/\gamma N_A(E_q/E_M-1)+E_q/E_Mn_++(E_D+iE_0)/E_M]^2-2(E_D+iE_0)/E_M[2(sI_0+\beta)/\gamma N_A+n_+]}, \tag{30}$$

$$\alpha_\pm=\frac{\gamma N_A}{2a}\Bigg[\frac{E_D+iE_0}{E_M}+\frac{E_q}{E_M}n_-+\frac{3a}{\gamma N_A} \pm\sqrt{[E_q/E_Mn_-+a/\gamma N_A+(E_D+iE_0)/E_M]^2-4(E_D+iE_0)/E_M(sI_0+\beta+\gamma N_An_+)/\gamma N_A}\Bigg], \tag{31}$$

$$C=-\frac{E_q}{E_M}\frac{a\{c_1E_M[sI_0+\beta)(n_++n_-)/2\gamma N_An_+n_-]+isI_0n_+E_q[2N_D/N_A-1+(sI_0+\beta)/\gamma N_A]\}}{(sI_0+\beta+n_+\gamma N_A)[an_+E_q+(sI_0+\beta+n_+\gamma N_A)(E_D+iE_0)]}. \tag{32}$$

IV. RESULTS OF THE SIMULATIONS AND DISCUSSION

From the general analytic solution (24) we can study any particular regime of interest. In this paper, we simulate the transient behavior of the space-charge field fundamental amplitude in a dark regime, where the grating is previously written with a two-wave interfering short pulse. Then the evolution, i.e., the formation and the decay of the photorefractive field amplitude, is analyzed beginning at $t=t_0$ just when the recorded pulse ends. The pulse excitation setup takes into account the values of E_1 and dE_1/dt at initial time t_0. These values are obtained from expression (24) by considering the recording intensity during the pulse temporal width τ_p. In our simulations we assume $\tau_p=10$ ns, and BSO typical parameters.[9,25–28] The analysis is done by varying the main excitation parameters: the pulse writing average intensity I_0, the grating spacing Λ, and the external field E_0, within the usual experimental range. The thermal re-excitation process is considered through the parameter β. When the thermal excitation is neglected $(\beta=0)$, and $t=t_0$ then $\xi=1$. As a consequence, the general solution (24) is not valid under this approach. Fortunately, the transient behavior of E_1 can also be derived in this case (see Appendix). Thus, it allows to compare the model predictions either by considering or by disregarding the re-excitation process of the photorefractive centers. Figure 1 shows the evolution of the space-charge field amplitude for three different levels of the writing average intensity. These three levels produce three different initial values of the free charge carrier density. The curves in each plot represent the evolution at different values of the thermal excitation rate β. The curve with $\beta=0$ is obtained from expression (A7). As expected, the decay time of the space field increases when β decreases, remaining a permanent grating when $\beta=0$ for $t\to\infty$. In Fig. 1(a), the writing pulse generates a low-free charge-carrier density of value $n_0=0.01N_A$. The nonlinear recombination of charge carriers is not significant in this regime.[8–10] We can observe that E_1 increases due to the diffusion current and reaches a steady state at maximum of the field value. If the free carriers are completely recombined in traps and cannot be re-excited $(\Gamma=0)$, then the permanent grating remains at this maximun value. However, this is an ideal situation. The actual grating decays to zero in the dark due to thermal erasure as the curves with $\beta\neq 0$ clearly show. In this situation the decay time is strongly dependent on thermal erasure rate. Figure 1(a) shows that the highest value of E_1 also depends on the erasure rate for significant values of β. The maximum value decreases when the erasure rate increases. In addition, it is verified that these decays can be fitted by exponential functions as Ref. 8 establishes. Figures 1(b) and 1(c), show a different behavior for E_1 when writing pulses of higher intensity are applied. When $n_0\ll N_A$ is not valid, the nonlinear effects become significant.[9,10] The writing intensity pulse generates a free charge carrier density of $0.16N_A$ in Fig. 1(b), and of $0.88N_A$ in Fig. 1(c). In Fig. 1(a) the curves show a depression. This depression was experimentally observed in several materials[9,11,13,32] and was theoretically predicted by using the band transport model[9,25] and also from the hopping

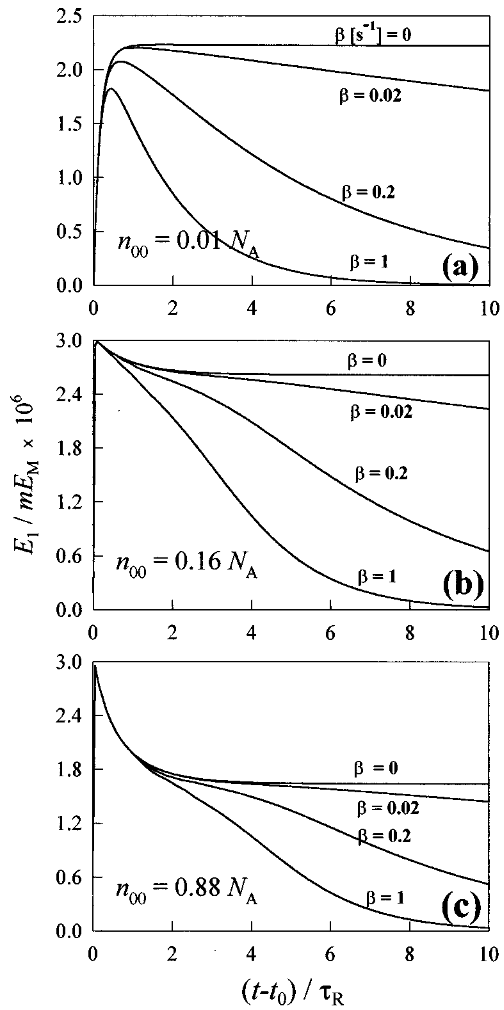

FIG. 1. Temporal evolution of the normalized space-charge field fundamental amplitude at several ranges of pulse intensities. I_0[W/cm^2]$=5\times10^3$ (a), 8×10^4 (b), and 5×10^5 (c). n_{00} indicates the initial free charge-carrier density in terms of N_A. The grating spacing is $\Lambda=10$ μm. The material parameters for all the figures are as in Refs. 9, and 16–18 simulating a BSO crystal. No external field is applied. $\tau_R=1/\gamma N_A$.

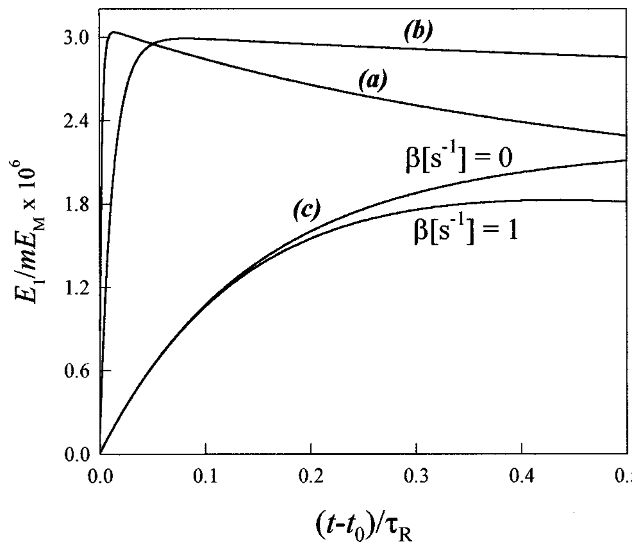

FIG. 2. The formation of the normalized space field is shown. Curves *a*, *b*, and *c*, correspond to the pulse intensities of Fig. 1. Curves *a* and *b* do not depend on thermal rate value, whereas curve *c* is plotted for two different β.

model.[30] However, the solutions obtained from numerical methods do not completely explain this phenomenon. This depression can be analyzed from Figs. 1(b) and 1(c) by observing the behavior of dE_1/dt, which is related with the current density. After the pulse ends, the diffusion current originates and the space-charge field increases ($dE_1/dt>0$). The maximum value of this field is reached when the drift current evens the diffusion current of opposite sign, and the net charge transport becomes null ($dE_1/dt=0$). Then, the space-charge field begins to decrease because the drift current advances the diffusion current and the net carrier transport reverses its direction ($dE_1/dt<0$). The depression depth indicates the importance of the drift transport mechanism and it increases when I_0 does, as Figs. 1(b) and 1(c) show. Therefore, the drift transport dominates the high-pulse excitation range, where the nonlinear effects are more strongly manifested. Finally, it is important to recall that this predicted depression at high-writing intensities does not depend on free carrier re-excitation processes. On the other hand, the curves in Figs. 1(b) and 1(c) could not be fitted by exponential functions. It confirms that this early band transport model predicts nonexponential decays at high-excitation intensities. In Figs. 1(b) and 1(c) a concavity change is displayed and it is more remarkable in Fig. 1(c) at highest pulses. This change takes place in the temporal range where the nonlinear recombination and the thermal re-excitation processes are strongly overlapped. The concavity change was experimentally observed.[9,10] On the other hand, the formation of E_1 for several writing pulse values is depicted in Fig. 2. By comparing different curves, it is apparent that the grating formation time decreases when the writing intensity increases. It is verified that the curves *a*, and *b*, where N_A is smaller than n_0, are independent of the β value. Then, the re-excitation process is not important in the formation of the space field for high values of the writing intensity. However, this could be not true for weak writing pulses. As Fig. 2 shows, the curve *c* ($n_0 \ll N_A$) depends on the erasure rate value. On the other hand, the expression (24) should permit the control of the spatial bandwidth that allows to store a holographic image by analyzing the grating response time in terms of the spatial grating period. Figure 3 shows that the grating rise time decreases [Fig. 3(a)] and the decay time increases [Fig. 3(b)] when the grating spacing Λ diminishes. The physical picture of this phenomenon can be explained as follows. Diffusion leads to the redistribution of the charge carriers and to a decreasing of the modulation of the charge-carrier grating. It is well known that the diffusion time is directly proportional to the grating spacing.[2] For small grating spacing, the diffusion time becomes either equal or smaller than the trapping time producing a homogeneous distribution of the charge carriers before they are trapped. In this case, the space-charge field amplitude is an inhomogeneous distribution of photoionized donors and the build-up time of the grating is given by the shorter diffusion time. On the contrary, for longer grating spacing the diffusion time becomes longer than the nonvariant lifetime of free electrons or holes. Then, the buildup of the grating is stopped because

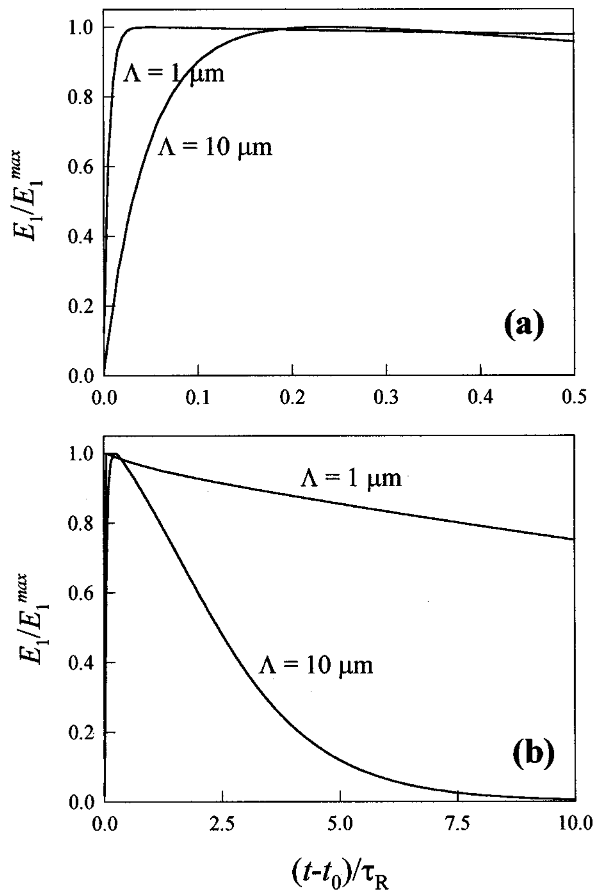

FIG. 3. Dependence of the photorefractive field evolution on grating spacing. The field is normalized at maximum value. (a) and (b) describe the formation and the decay of the field, respectively, for $\beta[\mathrm{s}^{-1}]=1$. The pulse intensity is $I_0[\mathrm{W/cm}^2]=2\times10^4$ and produces an $n_{00}=0.04N_A$. No external field is applied.

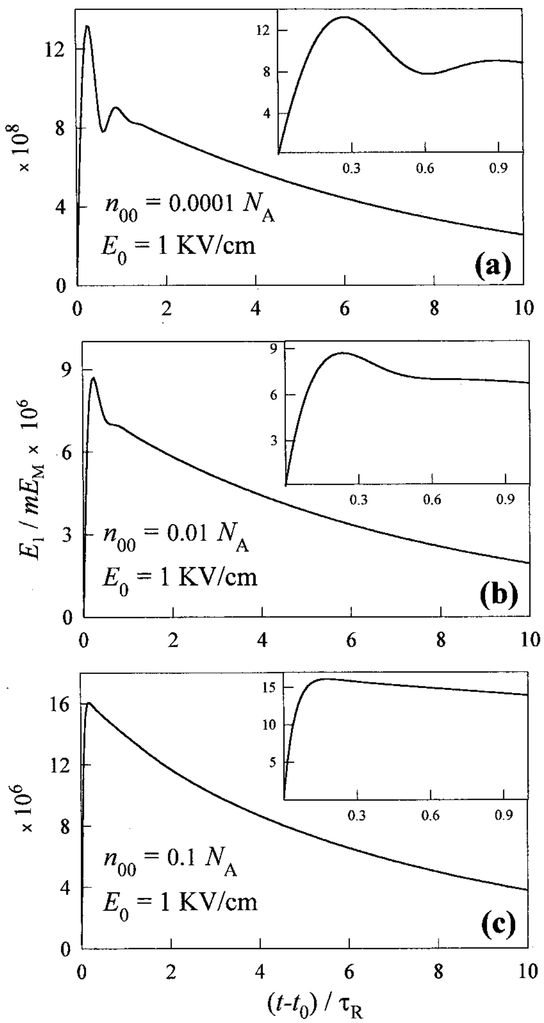

FIG. 4. Normalized space-charge field evolution with an applied external field at several ranges of initial charge densities. The thermal rate is $\beta[\mathrm{s}^{-1}]=1$ and the grating spacing is $\Lambda=10\ \mu$m. The insert figure describes the oscillatory behavior.

the photoexcited carriers are trapped within a distance much smaller than the grating spacing. Therefore, the grating is completely built in a period of time determined by the larger between free electron and hole lifetime. The re-excitation, and the transport of the charge-carrier density average to its thermal equilibrium value (determined by the dark conductivity) control the decay time. This relaxation process is lower for smaller grating spacing. In fact, the built internal field drifts the re-excited charge carriers at equilibrium. This field opposes to the diffusion field that diminishes for larger grating spacing. Thus the field resulting on the carriers is stronger producing a higher drift rate for larger spacing. As a consequence, a fast returning to equilibrium and therefore a smaller storage time at higher Λ values is produced. These results agree with measurements in grating build-up time[11,13] and grating decay time.[10,33] Also, from the general solution (24), the influence of an external field on grating behavior is studied. The damped oscillatory response of the space-charge field, experimentally observed,[12,34] is analyzed when a dc electric field is applied. This field adds a nonbalanced component to the drift current that competes with both the diffusion and the internal drift field. This nonbalanced factor gives rise to an oscillatory component of the grating when the drift current advances the diffusion current and vice versa. This phenomenon repeats itself. The oscillations are damped going to a thermal equilibrium value because the drift component due to the internal field is reduced after each oscillation. Figure 4 shows the transient behavior of the grating for a fixed external field and several writing intensity pulses. We can see that the oscillation amplitude depends strongly on initial carrier density, i.e., on writing excitation intensity. Figures 4(a), 4(b), and 4(c) show that the amplitude decreases as the writing pulse intensity increases and the oscillatory behavior can be completely screened at considerable pulse intensity [Fig. 4(c)]. At large average excitation intensity, the internal space-charge field becomes significant with respect to the external field. In consequence there exist regions in which the free carrier nonstationary current should be impeded reducing the transient oscillations. Also, it is verified that this behavior holds when the applied field value is changed. On the other hand, the increasing of the external field produces a higher nonbalanced free charge-carriers movement. Due to this fact, the frequency increases in the oscillatory component of the space-charge field as Fig. 5 clearly shows. Also, from Fig. 5 it is clear that the enhancement of the unstable component with stronger dc field precludes the free charge carriers going to equilibrium, decreasing the grating decay time.

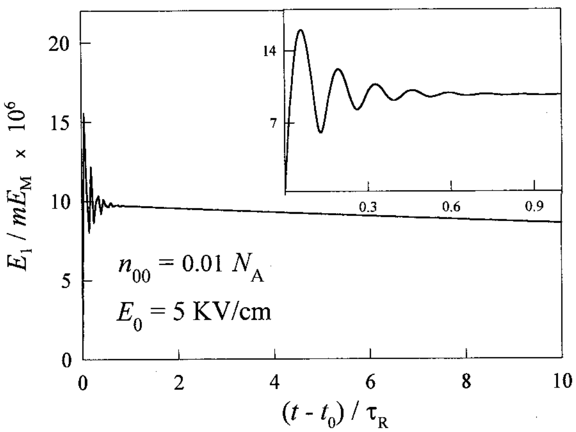

FIG. 5. Normalized space-charge field evolution with an applied external field of 5 KV/cm. The thermal rate is $\beta[s^{-1}]=1$ and the grating spacing is $\Lambda=10\ \mu$m.

V. CONCLUSIONS

The transient behavior of the fundamental amplitude of the space-charge field has been derived in analytical general form within the band transport model that involves the nonlinear recombination and the thermal re-excitation of the photorefractive centers. Based on this solution, different regimes of interest can be rigorously studied. In particular, we have simulated the temporal evolution of the photorefractive field under previous writing pulse conditions. The formation and decay of the photorefractive field in different range of excitation parameters have been studied. Our predictions are in agreement with experimental results obtained in several works. The behavior of the field is different at several ranges of the writing intensity pulse. At high intensities, a field decay that does not depend on erasure rate is predicted. For this regime, where N_A is smaller than n_0, the strong nonexponential decay is also verified. The overlapping of recombination and re-excitation processes is apparent by observing the concavity change in the space-charge field decay. The formation and decay time of the photorefractive field depend on the grating spacing. As the spacing Λ increases, the rise time diminishes and the decay time increases. Also, the oscillatory transient behavior with an applied field has been analyzed. When the applied field increases the frequency of its oscillations increases while the amplitude decreases. Moreover, the oscillatory behavior can be completely screened using adequate values of writing pulse. Finally, the model predicts the increasing of the grating decay time when the applied field increases.

APPENDIX

If we analyze the transient behavior of the space-charge field in the dark by disregarding the re-excitation process of the photorefractive centers ($\beta=0$), then the expression (24) is not valid since $\xi=1$ at $t=t_0$. Therefore we must find the general solution for the space-charge field in this particular case where Eq. (7) becomes

$$\frac{d^2E_1^*}{dt^2}+\left[\gamma N_A\left(1+\frac{E_D+iE_0}{E_M}\right)+(\gamma_0+2\gamma)n_0^*(t)\right]\frac{dE_1^*}{dt}+\left[\gamma^2N_A\left(\frac{E_D+iE_0}{E_M}\right)n_0^*(t)+\gamma_0\gamma[n_0^*(t)]^2\right]E_1^*=0, \tag{A1}$$

with the evolution of the free charge carriers given by[24]

$$n_0^*(t)=N_A\frac{n_{00}\exp(-t/\tau_R)}{N_A+n_{00}-n_{00}\exp(-t/\tau_R)}. \tag{A2}$$

Equation (A1) can also be transformed in a hypergeometric type equation by means of the following steps. First, we define an auxiliary variable ξ^* as

$$\xi^*=1-\frac{n_{00}}{1+n_{00}}\exp(-t/\tau_R), \tag{A3}$$

and introduce an auxiliary function W_1^* in terms of ξ^* with the condition

$$W_1^*(\xi^*)=E_1^*(t). \tag{A4}$$

Then we apply the following transformation

$$W_1^*=(\xi^*)^{E_q/E_M}W_2^* \tag{A5}$$

to finally obtain the hypergeometric equation[31]

$$\xi^*(1-\xi^*)\frac{d^2W_2}{(d\xi^*)^2}+\left[\left(2-\frac{E_q}{E_M}\right)(1-\xi^*)+\left(\frac{E_D+iE_0}{E_M}\right)\xi^*\right]\frac{dW_2^*}{d\xi^*}+\left(1-\frac{E_q}{E_M}\right)\left(\frac{E_D+iE_0}{E_M}\right)W_2^*=0. \tag{A6}$$

The general solution of Eq. (A6) is

$$E_1^*(t)=mE_M\left[A_1^*(\xi^*)^{E_q/E_M}F\left(1-\frac{E_q}{E_M},-\frac{E_D+iE_0}{E_M};1-\frac{E_q}{E_M};1-\frac{n_{00}}{1+n_{00}}\times\exp(-t/\tau_R)\right)+A_2^*(\xi^*)^{-1}\right], \tag{A7}$$

where A_i^* are constants determined by initial conditions.

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