It is possible that each light sensor pixel in the eye has the capability of measuring the distance to the part of the object in focus at the pixel. One can also construct an electronic camera where each pixel can measure the distance to the portion of the object in focus at the pixel. That is, these devices have depth perception.

It is possible that each light sensor pixel in the eye has the capability of measuring the distance to the part of the object in focus at the pixel. The rods and cones which are actually cell cilia or flagella that act as short lightguides. It is these cilia that facilitate the depth perception. One can also

![Diagram of Light Waves and Depth Perception](image)

**Fig. 1.** A group of lightwaves of different wavelength will all have the same phase at the image as they had at the object where the light was emitted. Thus, if
the wavelength and phase of each component of the light is known at the image the distance L to the object can be determined.

construct an electronic camera where each pixel can measure the distance to the portion of the object in focus at the pixel, that is, build a camera with depth perception.

The device functions as follows: A group of lightwaves of different wavelength as shown in Fig. 1 will all have the same phase at the image as they had at the object where the light was emitted. Thus, if the wavelength and phase of each component of the light is known at the image the distance L to the object can be determined. However, some kind of guidance mechanism is required for the lightwaves that were emitted by an object point to be guided to the image point. This can readily be achieved with an ordinary convex lens. Indeed, cameras and animal eyes have been using lenses very successfully for this purpose. Light takes the same time to traverse any path from an object point through the lens to an image point. Neglecting aberrations of the lens, by adding all the lightwave components of different wavelength at an image point one obtains an image if all the lightwave components have the same phase as they had at the object.

All this is also true if the light is only partially coherent. Coherence can be illustrated by two light beams originating at the same time from the same object point that travel over different path length to a single image point. The lightwaves will form an interference pattern if the difference in the length of the two light path is less than the coherence length of the light. If the difference

![Diagram](image)

**Fig. 2.** A simple apparatus that is both sensitive to the wavelength and phase of the arriving light is a short piece of lightguide. In order to obtain the wavelength and phase information of the light components measurements of a number of modes of the lightguide are made.
path length is longer than the coherence length of the lightwaves the light will form a more or less uniform bright spot when observed over a length of time. All lasers have some finite coherence length. Even white light has a coherence length of a few wavelengths. If this were not so lenses would not work.

The distance from an image point to its object point can be determined from the phase of a number of components of light at different wavelengths even if the light is only coherent for a distance of a few wavelengths.

In order to determine the distance from each point of an image to its corresponding object point one needs a simple apparatus at each image point that can measure the phase and wavelength of each component of the light. A short resonant waveguide section as shown in Fig. 2 with an detector array at its far end is capable of both detecting wavelength and phase. The detector array can detect the various modes in the lightguide.

The information measured by the light sensors at the end of the lightguide also contains the brightness of the light radiated by the object point. To eliminate the brightness information and retain the phase information measurements of at least two lightguide modes are made by an detector array at the far end of the lightguide. Thus there has to be a light sensor array at the far end of the lightguide capable of distinguishing between the modes in the lightguide.

![Diagram](image.png)

**Fig. 3.** Single pixel imaging sensor.
The partially coherence of the light is taken in consideration in this calculation. To accomplish this the correlation between parts of the light waves has to be performed. Fortunately the light intensity is expressed as the product of the electric field and magnetic flux density of the light electromagnetic field. Thus a correlation between the light electric field and the light magnetic flux density can be performed. In the mathematical model, low frequencies corresponding to long wavelength give the largest interaction terms with long distances. Light wavelength are much smaller than the distances to be measured. However, again, since the correlation between the light electric field and light magnetic flux density is calculated the sum and difference frequencies appear. It is the low difference frequencies corresponding to long wavelengths that give the largest interaction terms.

First one has to calculate the light electric field at the input of the lightguide. Since in this part of the problem all distances are much larger than a light wavelength a scalar diffraction theory model can be used. Also, the paraxial Fresnell approximation is used in this calculation. The scalar light electric field $u_1(r_1)$ at the input plane of the lens is:

$$u_1(r_1) = \frac{\omega}{j2\pi ca} \exp\left[ j\frac{\omega a}{c} \int_{S_o} ds_o \ u_o(r_o) \exp\left[ \frac{\omega}{2ca} |r_1 - r_o|^2 \right] \right] \tag{1.1}$$

where $u_o(r_o)$ is the scalar light electric field at the object and the integral is over the object plane $S_o$. The scalar light electric field $u_2(r_1)$ at the output plane of the lens is:

$$u_2(r_1) = u_1(r_1) \exp\left[ -\frac{\omega r_1^2}{2cf} \right] \tag{1.2}$$

The scalar light electric field $u_3(r_2)$ at the input to the lightguide section is:

$$u_3(r_2) = \frac{\omega}{j2\pi cb} \exp\left[ j\frac{\omega n_1}{c} \int_{S_1} ds_1 \ u_2(r_1) \exp\left[ \frac{\omega n_1}{2cb} |r_2 - r_1|^2 \right] \right] \tag{1.3}$$

where $n_1$ is the index of refraction of the material in the eye and the integral is over the pupil of the eye or camera. By substituting equation 1.1 into equation 1.2 and the resulting expression into equation 1.3 one obtains:
\[ u_3(r_2) = -\frac{\omega^2}{4\pi^2c^2ab} \exp \left( j\frac{\omega(a + bn_1)}{c} \right) \int_{S_0} ds_o \int_{S_1} ds_1 u_0(r_o) \exp \left[ \frac{j\omega(\frac{r_1^2 - 2r_1 \cdot r_2 + r_2^2}{2ca})}{2c} \right] \exp \left( -\frac{\omega_{n_1}(r_2 - 2r_1 \cdot r_2)}{2cb} \right) \]

(1.4)

The image will be in focus on the object if:

\[ \frac{1}{a} + \frac{n_1}{b} - \frac{1}{f} = 0 \]  

(1.5)

By substituting equation 1.5 into equation 1.4 one obtains:

\[ u_3(r_2) = -\frac{\omega^2}{4\pi^2c^2ab} \exp \left( j\frac{\omega(a + bn_1)}{c} \right) \int_{S_0} ds_o u_0(r_o) \exp \left[ j\frac{\omega\left(\frac{r_o^2}{a} + \frac{n_1 r_2^2}{b}\right)}{2ca} \right] \times \]

\[ \int_{S_1} ds_1 \exp \left[ -j\frac{\omega}{ca} r_1 \left( r_o + \frac{an_1 b}{r_2} \right) \right] \]

(1.6)

Since the pupil diameter is much larger than a light wavelength the integral over \( S_1 \), the pupil, can be approximated by a delta function.

\[ \int_{S_1} ds_1 \exp \left[ -j\frac{\omega}{ca} r_1 \left( r_o + \frac{an_1 b}{r_2} \right) \right] - 4\pi^2 c^2 \delta \left( r_o + \frac{an_1 b}{r_2} \right) \]

(1.7)

By substituting equation 1.7 into equation 1.6 one obtains:

\[ u_3(r_2) = -\frac{\omega}{b} \exp \left( j\frac{\omega(a + bn_1)}{c} \right) \int_{S_0} ds_o u_0(r_o) \exp \left[ j\frac{\omega\left(\frac{r_o^2}{a} + \frac{n_1 r_2^2}{b}\right)}{2ca} \right] \delta \left( r_o + \frac{an_1 b}{r_2} \right) \]

(1.8)

By integrating one obtains for the light electric field at the input to the light guide:
\[ u_3(\mathbf{r}_2) = -\frac{a}{b} u_0 \left( -\frac{a n_1}{b} r_2 \right) \exp \left\{ j \frac{\omega}{c} \left[ a + n_1 b + \left( 1 + \frac{n_1 a}{b} \right) \frac{n_1 r_2^2}{2b} \right] \right\} \tag{1.9} \]

The amplitude the light electric field at the entrance to the lightguide and the effective distance \( L \) from the object to the lightguide entrance is:

\[ a) \quad E_o \equiv -\frac{a}{b} u_0 \left( \frac{a n_1}{b} \mathbf{r}_2 \right) \quad b) \quad L = a + n_1 b + \left( 1 + \frac{n_1 a}{b} \right) \frac{n_1 r_2^2}{2b} \tag{1.10} \]

Next, the propagation of the light through the lightguide is calculated. Here the scalar paraxial approximation is no longer valid. In order to include the fact that the light has a limited coherence time \( \eta \) the light electric field vector in the air in front of the lightguide is formulated as a random process: The light electric field entering the light guide is a component of the expansion of the light electric field of equation 1.9 in the modes of the light guide.

\[ E_1 = \hat{a_m} E_o \exp[ j m \phi] S(r) \left\{ \exp \left[ j \left( \omega + \frac{\theta}{\eta} \right) \left( t - \frac{z}{c} \right) \right] + \text{Re} \exp \left[ j \left( \omega + \frac{\theta}{\eta} \right) \left( t + \frac{z}{c} \right) \right] \right\} \tag{1.11} \]

where it was assumed that some light is reflected at the entrance to the lightguide. Here \( \omega \) is the oscillating frequency of the light, \( m \) is an integer, \( \eta \) is a coherence time and \( \theta \) is uniformly distributed random phase. The probability density \( p(\theta) \) of the random phase is:

\[ p(\theta) = \frac{1}{2\pi} \quad \text{for} \quad -\pi < \theta \leq \pi \tag{1.12} \]

For simplicity TE like modes are assumed. The corresponding magnetic flux density pseudo vector component in the air between the object and the light guide can be obtained by substituting the electric field into the Faraday Maxwell equation. The magnetic flux density pseudo vector has also the form of a random process.
\[ B_1 = -\frac{E_0}{c} a_r \exp \{ jm\phi \} S(r) \left\{ \exp \left[ j \left( \frac{\omega}{\eta} \right) \left( t - \frac{n(z - L)}{c} \right) \right] - R \exp \left[ j \left( \frac{\omega}{\eta} \right) \left( t + \frac{n(z - L)}{c} \right) \right] \right\} + \\
\quad j a_z \frac{E_0}{c} \exp \{ jm\phi \} \frac{1}{Z} r S(r) \left\{ \exp \left[ j \left( \frac{\omega}{\eta} \right) \left( t - \frac{n(z - L)}{c} \right) \right] + R \exp \left[ j \left( \frac{\omega}{\eta} \right) \left( t + \frac{n(z - L)}{c} \right) \right] \right\} \] 

(1.13)

Here the function \( S(r) \) of the transverse coordinate \( r \) can be of the following form for a fiber like lightguide.

\[ S(r) = J_m (r q_m) \]

(1.14)

where \( J_m (r q_m) \) is the \( m \)'th order Bessel function. The light electric field vector in the cilia lightguide is:

\[ E_2 = a_r E_0 S(r) \exp \{ jm\phi \} \left\{ F \exp \left[ j \left( \frac{\omega}{\eta} \right) \left( t - \frac{n(z - L)}{c} \right) \right] + G \exp \left[ j \left( \frac{\omega}{\eta} \right) \left( t + \frac{n(z - L)}{c} \right) \right] \right\} \]

(1.15)

The corresponding magnetic flux density pseudo vector in the cilia lightguide, similar to equation 1.3 is:

\[ B_2 = -\frac{a_r E_0}{c} \exp \{ jm\phi \} S(r) \left\{ F \exp \left[ j \left( \frac{\omega}{\eta} \right) \left( t - \frac{n(z - L)}{c} \right) \right] - \\
\quad G \exp \left[ j \left( \frac{\omega}{\eta} \right) \left( t + \frac{n(z - L)}{c} \right) \right] \right\} + \\
\quad j a_z E_0 \exp \{ jm\phi \} \frac{1}{Z} r S(r) \left\{ F \exp \left[ j \left( \frac{\omega}{\eta} \right) \left( t - \frac{n(z - L)}{c} \right) \right] + \\
\quad G \exp \left[ j \left( \frac{\omega}{\eta} \right) \left( t + \frac{n(z - L)}{c} \right) \right] \right\} \]

(1.16)

where the dimension less quantity \( n \) is the mode dependent effective index of refraction of the lightguide. The dispersion relation of the light guide.
\[ q_m^2 + \frac{2 \omega^2}{n^2} = \frac{\omega n^2_2}{c^2} \]  

(1.17)

and where \( q_m \) is the transverse \( m \) dependent mode wave vector, see equation 1.14. The light electric field vector in the detector is:

\[
E_3 = a' \omega E_0 S(r) \exp\left\{ j \left( \omega + \frac{\theta}{\eta} \right) \left[ t - \frac{(\beta - j\alpha)(z - L - b)}{c} \right] \right\} 
\]

(1.18)

Again, similar to equation 1.13 the corresponding magnetic flux density pseudo vector in the detector is:

\[
B_3 = \frac{1}{c} E_0 (\beta - j\alpha) \exp\left\{ j m \phi \right\} S(r) \text{exp}\left\{ \left( \omega + \frac{\theta}{\eta} \right) \left[ t - \frac{(\beta - j\alpha)(z - L - b)}{c} \right] \right\} + \\
ja_z E_0 \exp\left\{ jm \phi \right\} \frac{1}{\omega \epsilon_0} \frac{Z}{Z_r} [S(r)] \text{exp}\left\{ \left( \omega + \frac{\theta}{\eta} \right) \left[ t - \frac{(\beta - j\alpha)(z - L - b)}{c} \right] \right\}
\]

(1.19)

The dispersion relation in the detector has the following form:

\[
q_m^2 + \frac{\omega^2 (\beta - j\alpha)^2}{c^2} = \frac{\omega n^2_3}{c^2} + j\omega \sigma \mu_o \]  

(1.20)

where \( \sigma \) is the effective conductivity at the light frequency. By subtracting equation 1.17 from equation 1.20 and multiplying by \( \frac{c^2}{\omega^2} \) one obtains for the real and imaginary parts:

a) \( \beta^2 - n^2 - n^2_3 + n^2_2 - \alpha^2 = 0 \)  

b) \( 2\alpha \beta = \frac{\sigma}{\omega \epsilon_o} \)  

(1.21)

By substituting equation 1.21b into equation 1.21a for \( \beta \) and solving the resulting equation for \( \beta \) one obtains:
\[ \beta = \sqrt{\frac{n^2 + n_3^2 - n_2^2}{2}} + \frac{1}{n^2} + \sqrt{1 + \frac{\sigma^2}{\omega^2 \varepsilon_o (n^2 + n_3^2 - n_2^2)^2}} \]  

(1.22)

An expression for \( a \) can be obtained by substituting equation 1.22 into equation 1.21b.

\[ \alpha = \frac{\sigma}{2\omega \varepsilon_o} \sqrt{\frac{n^2 + n_3^2 - n_2^2}{2}} + \frac{1}{n^2} + \frac{\sigma^2}{\omega^2 \varepsilon_o (n^2 + n_3^2 - n_2^2)^2} \]  

(1.23)

**BOUNDARY CONDITIONS**

At \( z = L \) the tangential electric field is continuous:

\[
\exp \left[ -j \left( \omega + \frac{\theta}{\eta} \right) \frac{L}{c} \right] + \Re \exp \left[ j \left( \omega + \frac{\theta}{\eta} \right) \frac{L}{c} \right] = F + G
\]

(1.24)

At \( z = L \) the perpendicular component, the \( r \) component, of the magnetic flux density pseudo vector is continuous:

\[
\exp \left[ -j \left( \omega + \frac{\theta}{\eta} \right) \frac{L}{c} \right] - \Re \exp \left[ j \left( \omega + \frac{\theta}{\eta} \right) \frac{L}{c} \right] = n(F - G)
\]

(1.25)

It is true that the tangential component of the magnetic flux density pseudo vector is also continuous at the boundary. However, it yields the same information as the one obtained from the continuity of the tangential electric field, equation 1.23. By adding equations 1.24 and 1.25 and solving for \( F \) one obtains:

\[
F = \frac{1}{2} \left\{ \left( 1 - \frac{1}{n} \right) \exp \left[ -j \left( \omega + \frac{\theta}{\eta} \right) \frac{L}{c} \right] + \Re \left( 1 - \frac{1}{n} \right) \exp \left[ j \left( \omega + \frac{\theta}{\eta} \right) \frac{L}{c} \right] \right\}
\]

(1.26)
By subtracting equation 1.25 from equation 1.24 and solving for $G$ one obtains:

$$G = \frac{1}{2} \left\{ \left( 1 - \frac{1}{n} \right) \exp \left[ -j \left( \omega + \frac{\theta}{\eta} \right) \frac{L}{c} \right] + R \left( 1 + \frac{1}{n} \right) \exp \left[ j \left( \omega + \frac{\theta}{\eta} \right) \frac{L}{c} \right] \right\}$$ (1.27)

At $z = L + b$ the tangential component of the electric field vector is continuous;

$$F \exp \left[ -j \left( \omega + \frac{\theta}{\eta} \right) \frac{nb}{c} \right] + G \exp \left[ j \left( \omega + \frac{\theta}{\eta} \right) \frac{nb}{c} \right] = T$$ (1.28)

At $z = L + b$ the perpendicular component, the $r$ component, of the magnetic flux density pseudo vector is continuous.

$$n \left\{ F \exp \left[ -j \left( \omega + \frac{\theta}{\eta} \right) \frac{nb}{c} \right] - G \exp \left[ j \left( \omega + \frac{\theta}{\eta} \right) \frac{nb}{c} \right] \right\} = (\beta - j\alpha) T$$ (1.29)

Adding equations 1.28 and 1.29 and solving for $F$ one obtains:

$$F = \frac{1}{2} \left[ 1 + \frac{\beta - j\alpha}{n} \right] T \exp \left[ j \left( \omega + \frac{\theta}{\eta} \right) \frac{nb}{c} \right]$$ (1.30)

Subtracting equation 1.29 from 1.28 and solving for $G$ one obtains:

$$G = \frac{1}{2} \left[ 1 - \frac{\beta - j\alpha}{n} \right] T \exp \left[ -j \left( \omega + \frac{\theta}{\eta} \right) \frac{nb}{c} \right]$$ (1.31)

The constant $F$ can be eliminated by equating equations 1.26 and 1.30.

$$\left( 1 + \frac{1}{n} \right) \exp \left[ -j \left( \omega + \frac{\theta}{\eta} \right) \frac{L}{c} \right] + R \left( 1 - \frac{1}{n} \right) \exp \left[ j \left( \omega + \frac{\theta}{\eta} \right) \frac{L}{c} \right] = 
\left( 1 + \frac{\beta - j\alpha}{n} \right) T \exp \left[ j \left( \omega + \frac{\theta}{\eta} \right) \frac{nb}{c} \right]$$ (1.32)

The constant $G$ can be eliminated by equating equations 1.27 and 1.31.
\[
\left(1 - \frac{1}{n}\right) \exp \left[ -j \left( \omega + \frac{\theta}{\eta} \right) \frac{L}{c} \right] + R \left(1 + \frac{1}{n}\right) \exp \left[ j \left( \omega + \frac{\theta}{\eta} \right) \frac{L}{c} \right] = \\
\left(1 - \frac{\beta - j\alpha}{n}\right) \exp \left[ -j \left( \omega + \frac{\theta}{\eta} \right) \frac{nb}{c} \right]
\]

This leaves the constants R and T. One next solves equations 1.32 and 1.32 for R

\[
\frac{1 + \frac{\beta - j\alpha}{n}}{1 - \frac{1}{n}} \exp \left[ -j \left( \omega + \frac{\theta}{\eta} \right) \frac{L - nb}{c} \right] - \frac{1 + \frac{1}{n}}{1 - \frac{1}{n}} \exp \left[ -j \left( \omega + \frac{\theta}{\eta} \right) \frac{2L}{c} \right] = R
\]

The constant R can be eliminated by equating equations 2.34 and 2.35. By equating equations 2.24 and 2.35 and collecting terms one obtains:

\[
T \left\{ \left(1 + \frac{1}{n}\right)(n + \beta - j\alpha) \exp \left[ j \left( \omega + \frac{\theta}{\eta} \right) \frac{nb}{c} \right] - \left(1 - \frac{1}{n}\right)(n - \beta + j\alpha) \exp \left[ -j \left( \omega + \frac{\theta}{\eta} \right) \frac{nb}{c} \right] \right\} = \\
4n \exp \left[ -j \left( \omega + \frac{\theta}{\eta} \right) \frac{L}{c} \right]
\]

By solving for T one obtains:

\[
T = \frac{4 \exp \left[ -j \left( \omega + \frac{\theta}{\eta} \right) \frac{L}{c} \right]}{P(\omega) \exp \left[ j \left( \omega + \frac{\theta}{\eta} \right) \frac{nb}{c} \right] - Q(\omega) \exp \left[ -j \left( \omega + \frac{\theta}{\eta} \right) \frac{nb}{c} \right]}
\]

where:

a) \( P(\omega) = \left(1 + \frac{1}{n}\right)(n + \beta - j\alpha) \)

b) \( Q(\omega) = \left(1 - \frac{1}{n}\right)(n - \beta + j\alpha) \)
and where \( n, \beta, \) and \( \alpha \) are functions of \( \omega \).

The light detector response is proportional to the \( z \) component of the electromagnetic power density \( S_z \) or Pointing’s vector evaluated at \( z = L + b \).

\[
S_z = \frac{1}{\mu_0} E_{\phi T}(z = L + b) B_{rT}^*(z = L + b) \tag{1.39}
\]

where \( E_{\phi T} \) is the \( \phi \) component of the total electric field, the electric field vector component integrated over the bandwidth \( B_w \) of the impinging light and \( B_{rT} \) is the component of the total magnetic flux density pseudo vector, the magnetic flux density pseudo vector component integrated over the bandwidth \( B_w \) of the impinging light.

\[
a) E_{\phi T} = \frac{1}{B_w} \int_{\omega_o - \frac{B_w}{2}}^{\omega_o + \frac{B_w}{2}} E_3 \cdot \hat{a} \, d\omega \\
b) B_{rT} = \frac{1}{B_w} \int_{\omega_o - \frac{B_w}{2}}^{\omega_o + \frac{B_w}{2}} B_3 \cdot \hat{a} \, d\Omega \tag{1.40}
\]

By substituting equations 1.18, 1.19 into equations 1.40 and substituting the resulting expression and equation 1.37 into equation 1.39 one obtains.

\[
S_z = \frac{E_o^2}{z_o} \frac{16}{B_w^2} \int_{\omega_o - \frac{B_w}{2}}^{\omega_o + \frac{B_w}{2}} d\omega \int_{\omega_o - \frac{B_w}{2}}^{\omega_o + \frac{B_w}{2}} d\Omega (\beta + j\alpha) \exp \left[ j \left( \Omega - \omega \right) \frac{L}{c} \right] \left[ P(\omega)P^*(\Omega) \exp \left[ j \left( \omega - \Omega \right) \frac{nb}{c} \right] + Q(\omega)Q^*(\Omega) \exp \left[ -j \left( \omega - \Omega \right) \frac{nb}{c} \right] - Q(\omega)P^*(\Omega) \exp \left[ -j \left( \omega + \Omega + \frac{2\theta}{\eta} \right) \frac{nb}{c} \right] - P(\omega)Q^*(\Omega) \exp \left[ j \left( \omega + \Omega + \frac{2\theta}{\eta} \right) \frac{nb}{c} \right] \right]^{-1} \tag{1.41}
\]

where \( \omega_o \) is the center frequency and \( B_w \) is the bandwidth. Here \( P \) and \( Q \) are functions of \( \omega \) and \( P^* \) and \( Q^* \) are functions of \( \Omega \). Note that;

\[
z_o = \mu_0 c \tag{1.42}
\]

One can make the following transformation of variables that denote the sum and difference frequencies:
By inverting equations 1.40 one obtains:

\[ a) \omega = \frac{\xi + \Xi}{2} \quad b) \Omega = \frac{\Omega - \omega}{2} \]  

(1.44)

By substituting equations 1.43 into equation 1.41 one obtains:

\[
S_z = \frac{E_o^2}{Z_o} \frac{16}{B_w} \int_{2\omega_o - B_w}^{2\omega_o + B_w} d\xi \int_{-2\omega_o}^{2\omega_o} d\Xi (\beta + j\alpha) \exp \left[ j \frac{\xi L}{c} \right] \left\{ P(\omega)P^*(\Omega) \exp \left[ j \frac{\xi nb}{c} \right] + 
Q(\omega)Q^*(\Omega) \exp \left[ j \frac{\xi nb}{c} \right] - Q(\omega)P^*(\Omega) \exp \left[ j \left( \frac{2\beta}{\eta} \right) \frac{nb}{c} \right] - 
P(\omega)Q^*(\Omega) \exp \left[ j \left( \frac{2\beta}{\eta} \right) \frac{nb}{c} \right] \right\}^{-1}
\]

(1.45)

P and Q are functions of \( \omega \) and \( P^* \) and \( Q^* \) are functions of \( \Omega \). However the parameters \( n, \beta \) and \( \alpha \) do not change much with frequency \( \omega \). Thus one can approximate \( n, \beta, \alpha, P \) and \( Q \) by constants. The bandwidth \( B_w \) contains a very large number of cycles of the frequency \( \xi \).

Therefore the integral over \( \xi \) can be assumed to go from minus to plus infinity. In order to perform the integral over \( \xi \) the following transformation of variables is made:

\[ a) \quad z = \exp \left[ j \frac{\xi nb}{c} \right] \quad b) \quad \frac{dz}{B_w} = -j \frac{c}{nb} \frac{dz}{z} \]  

(1.46)

since the exponential is a constant \( z \) is a complex variable. Note that the magnitude of \( z \) is constant and equal to one. Thus equation 1.45 can be rewritten as follows:

\[
S_z = \frac{E_o^2}{Z_o} \frac{16c}{nbB_w} \int_{2\omega_o - B_w}^{2\omega_o + B_w} d\Xi \left( \beta + j\alpha \right) \int_{-2\omega_o}^{2\omega_o} \frac{dz}{z} \left( z - \frac{1}{nb} \right) \left( z - 2U(\Xi, \theta)z + \frac{QQ^*}{PP} \right) \]  

(1.47)

where
a) \[ 2U(\Xi, \theta) = \frac{1}{PP^*} \left\{ PQ^* \exp \left[ -j \left( \frac{\Xi + 2\theta}{\eta} \right) \frac{nb}{c} \right] + QP^* \exp \left[ j \left( \frac{\Xi + 2\theta}{\eta} \right) \frac{nb}{c} \right] \right\} \]

b) \[ 2U(\Xi, \theta) = \frac{2}{PP^*} \left( 1 - \frac{\beta^2 - \alpha^2}{n^2} \right) \left[ \frac{\left( n^2 - \beta^2 - \alpha^2 \right) \cos \left( \frac{\Xi + 2\theta}{\eta} \right) \frac{nb}{c} + 2\alpha \sin \left( \frac{\Xi + 2\theta}{\eta} \right) \frac{nb}{c} }{n} \right] \]

\[ (1.48) \]

For an effective object distance \( L \) equal to 1 m, a 10 µm long lightguide with an effective index of refraction \( n \) of 1.326885, \( \frac{L}{nb} \) is approximately equal to 75187. The denominator of equation 1.47 can be expended as follows:

\[ \frac{E_o}{z_o} \frac{16c}{nbB_w} \int_{2\omega_o - B_w}^{2\omega_o + B_w} \frac{d\Xi}{\beta + j\alpha} \int_{\text{Unit circle}} \frac{-\frac{L}{nb} dz}{(z - z_1)(z - z_2)} \]

\[ (1.49) \]

where:

a) \[ z_1 = U(\Xi, \theta) - \sqrt{U^2(\Xi, \theta) - \frac{QQ^*}{PP^*}} \] and

a) \[ z_2 = U(\Xi, \theta) + \sqrt{U^2(\Xi, \theta) - \frac{QQ^*}{PP^*}} \]

\[ (1.50) \]

Estimated values of \( z_1 \) and \( z_2 \) can be calculated from equation 1.48. They are \( z_1 \) is approximately equal to - 0.347861 and \( z_2 \) is approximately equal to - 6.35738 \times 10^{-5}. Since both \( z_1 \) and \( z_2 \) are less than 1 in magnitude equation 1.49 can be integrated by using the method of residues:

\[ \frac{E_o}{z_o} \frac{32\pi c}{nbB_w} \int_{2\omega_o - B_w}^{2\omega_o + B_w} \frac{d\Xi}{\beta + j\alpha} \frac{-\frac{L}{nb} - \frac{L}{nb}}{z_1 - z_2} \]

\[ (1.51) \]
**Fig. 4.** Plot of the ratio of the amplitude of the zero-order and first-order modes as a function of the distance to the object point that is in focus at the particular pixel. Spikes are quantization noise in the calculation.

Both $z_1$ and $z_2$ are functions of the random phase $\theta$. Thus the average value of Poynting's vector over the random phase $\theta$ has to be calculated next.

\[
<S_z> = j \frac{E_o^2}{z_0^2} \frac{16c}{nB} \int_{2\omega_o - B_w}^{2\omega_o + B_w} \int_{-\pi}^{\pi} d\xi d\theta \beta + j\alpha PP^* \frac{L}{Z_1 - Z_2} \frac{L}{Z_1^{nb} - Z_2^{nb}}
\]

(1.52)

The integral over $\theta$ has to be evaluated numerically.
In order to eliminate the amplitude term \( \frac{E^2}{z_0} \) and only retain the phase information the light power density \( <S_z> \) or Poynting's vector is measured for two different modes. The ratio of these two measurements is calculated. The amplitude term cancels in the ratio of the amplitudes of the two modes. The following parameters where used in the calculation:

\[
\begin{align*}
    n_2 &= 1.33 \\
    n_3 &= 3.5 \\
    \sigma &= 370000 \\
    J_0(aq_1) &= 0 \text{ at } aq_1 = 2.40825557 \\
    J_1(aq_1) &= 0 \text{ at } aq_1 = 3.83170597
\end{align*}
\]

where \( a = 2.5 \, \mu m \) is the radius of the lightguide section. A 10 \( \mu m \) long light guide was used.

A plot of the of the ratio of the amplitudes of the zero order and first order modes is shown in Fig. 4. Note that the amplitude ratio increases with the distance from the camera or eye pixel to the point on the object that is in focus at the pixel.

**APPENDIX A**

The curl in cylindrical coordinates is:

\[
\nabla \times \mathbf{F} = \hat{a}_r \left( \frac{1}{r} \frac{\partial}{\partial \phi} \frac{\partial F_z}{\partial \phi} - \frac{\partial F_r}{\partial z} \right) + \hat{a}_\phi \left( \frac{\partial F_z}{\partial \phi} - \frac{\partial F_z}{\partial r} \right) + \hat{a}_z \left[ \frac{1}{r} \frac{\partial}{\partial r} \left( r F_\phi \right) - \frac{1}{r} \frac{\partial F_\phi}{\partial r} \right] \tag{A.1}
\]

**REFERENCES**

1. The Stiles and Crowford effect shows the eye to be sensitive to the direction from which collimated white light beams enter the eye. First measurements were made by W. S. Stiles and B. H. Crawford in 1933.